$p(\Delta F^{\circ})$ = probability density of ΔF°
Y = dependent variable (random)
μ_X, μ_Y = ensemble mean of X, Y
σ_X, σ_Y = ensemble standard deviation of X, Y
σ_X^2, σ_Y^2 = ensemble variance of X, Y

Overlays

$\wedge$ = estimated improperly

LITERATURE CITED

Hougen, O. A., K. M. Watson, and R. A. Ragatz, *Chemical Process Principles, Part II-Thermodynamics*, p. 987, Wiley, New York (1959).

Manuscript received March 16, 1979; revision received May 25, and accepted May 30, 1979.

A Theoretical Criterion of Transition in the Free Motion of Single Bubbles and Drops

ISAAC H. LEHRER

Department of Chemical Engineering
Monash University
Clayton, Victoria 3168, Australia

Quantitative description of the behavior of bubble and drop dispersions in continuous fluids is often based on the description of free motion of a single bubble or drop, whose size is usually defined by the diameter D of the volume equivalent sphere; that is

$$D = (6V/\pi)^{1/3} \tag{1}$$

A coarse classification of the free motion of single bubbles and drops distinguishes between two main regimes (Figure 1). In regime 1, terminal velocity U is influenced by viscosity. In regime 2, terminal velocity U is predicted by equations that are independent of viscosity.

In a given combination of clean fluids, there is a critical diameter D_c such that particle motion is in regime 1 when $D < D_c$ and in regime 2 when $D > D_c$; the terminal velocity U_c of a bubble or drop of size D_c is a peak velocity (Figure 1).

A calculable criterion for transition from regime 1 to regime 2 is of interest. In a given combination of fluids, particle size is the independent variable; the problem is the prediction of D_c. Implicit and explicit empirical predictions of D_c have been made in conjunction with observation of liquid-liquid systems, for example, Hu and Kintner (1955), Klee and Treybal (1956), Krishna et al. (1959), Yamaguchi et al. (1975). There are empirical predictions for onset of bubble path oscillation (Tsuge and Hibino, 1977). In conjunction with other aspects, an ad hoc equation was formulated that predicts D_c in both bubble and drop systems (Lehrer, 1977). One form of this equation is

$$D_c = \frac{4.67\ \{(2\mu_d + \mu_c)/2\}^{1/3}}{\rho_c^{1/6}} \frac{\sigma^{1/4}}{(\Delta\rho\ g)^{5/12}} \tag{2}$$

While the predictive ability of Equation (2) compares favorably with that of others, it still lacks an analytical basis.

The basic equation for terminal velocity of a single particle is derived from the force balance

$$\pi D^3 \Delta\rho\ g/6 = C_D(\pi D^2/4)(\rho_c\ U^2/2) \tag{3}$$

A new transition criterion is based on the following considerations.

In regime 1, particle shape is almost spherical, and with increasing D, terminal velocity U becomes greater than that of rigid spheres of equal D that are subject to the same force $\Delta\rho V g$. The drag reduction has been ascribed to orderly internal circulation which, near the interface, moves parallel to the external flow. The wake is only narrow. Such flows have been observed in drop experiments (Kintner, 1963), with external streamlines resembling those of potential flow. In evaluation of the drag of spherical gas bubbles in liquids, Haas et al. (1972) showed that due to the assumption of a mobile interface, streamlines in the continuous phase almost coincide with those for potential flow.

In regime 2, particle shape is nonspherical and may oscillate strongly; motion may be irregular also. Periodic wakes follow air bubbles whose Re is of order 10^3 (Lindt, 1971), and drops move with vortex shedding (Davies, 1972).

In view of the foregoing, it is postulated here that at the transition from regime 1 to regime 2 in clean systems, the interface moves with the velocity v_θ of the tangential component of potential flow around a sphere. In regime 2, irregular and oscillating motion of the interface eliminates viscosity as a significant calculable factor.

Related to discussion of drag reduction, Ackeret (1952) derived an equation that states the power required to move a curved surface that is in contact with viscous fluid, the surface moving at the velocity v_θ of the tangential component of potential flow. The power is

$$P = -\int \tau_0 v_\theta\ dA \tag{4}$$

For the surface of a sphere of radius R in free stream velocity U

$$\tau_0 = 3\mu_c\, U\, (\sin\theta)/R \tag{5}$$

The power required to move the surface of the sphere at the velocity v_θ of the tangential component of potential flow around the sphere is also the rate of work against the drag force, so that with Equations (3), (4), (5) and the appropriate equations for v_θ and dA

$$P = 9\pi\mu_c R\, U^2 \int_0^\pi \sin^3\theta\, d\theta = 12\pi\mu_c R\, U^2 = C_D\, \pi R^2 \rho_c\, U^3/2 \tag{6}$$

From Equation (6)

$$C_D = \frac{24\,\mu_c}{\rho_c\, U\, R} = \frac{48\,\mu_c}{\rho_c\, U\, D} = \frac{48}{Re} \tag{7}$$

This is Ackeret's result (1952).

In the case of bubbles and drops at the transition point, there is motion both inside and outside of the interface. Power is required to move the interface at velocity v_θ in the presence of viscosity μ_c in the continuous phase and viscosity μ_d in the disperse phase. Applying Equation (6) to both sides of the interface, power is, with $R = D/2$

$$P = 6\pi(\mu_c + \mu_d)\, D_c\, U_c^2 \tag{8}$$

The concomitant drag coefficient is, with Equation (3)

$$C_D = \frac{48(\mu_c + \mu_d)}{\rho_c\, U_c\, D_c} = \frac{4\Delta\rho\, g\, D_c}{3\rho_c\, U_c^2} \tag{9}$$

From Equation (9), terminal velocity is

$$U_c = \frac{\Delta\rho\, g\, D_c^2}{36(\mu_c + \mu_d)} \tag{10}$$

In regime 2, terminal velocity is given by (Lehrer, 1976)

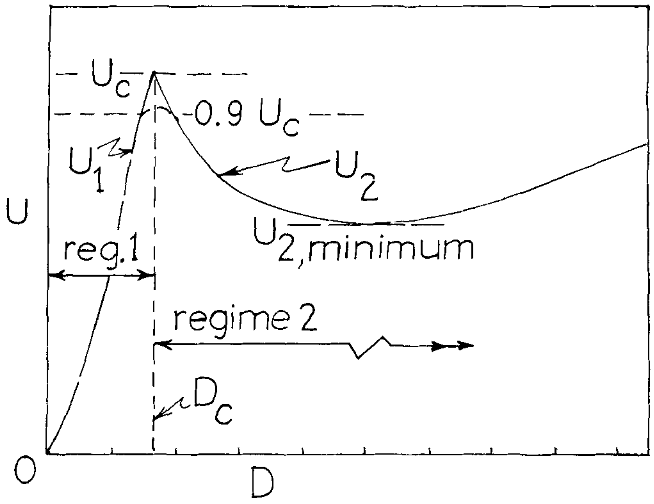

Figure 1. Terminal velocity *U* vs. bubble or drop diameter *D*, regimes and typical relation for clean fluids of low viscosity.

$$U_2 = \left(\frac{3\sigma}{\rho_c D} + \frac{\Delta\rho\, g\, D}{2\rho_c}\right)^{1/2} \tag{11}$$

At the transition point, U_2 equals U_c (Figure 1), so that from Equations (10) and (11)

$$U_c^2 = U_2^2 = \left(\frac{\Delta\rho\, g}{\mu_c + \mu_d}\right)^2 \frac{D_c^4}{1\,296} = \frac{3\sigma}{\rho_c D_c} + \frac{\Delta\rho\, g\, D_c}{2\rho_c} \tag{12}$$

and from Equation (12)

$$D_c^5 = \frac{648\,\sigma}{\rho_c}\left(\frac{\mu_c + \mu_d}{\Delta\rho\, g}\right)^2 (6 + E\ddot{o}) \tag{13}$$

Table 1. Observed and Predicted Values of Critical Bubble or Drop Diameter D_c

System: Dispersed phase	Continuous phase	Range of observed diameter D in which $U > 0.9\,U_c$ = range of D_c, mm	Predicted D_c: Minimum Equation (14), mm	Predicted D_c: Maximum Equation (17), mm
Drops in liquid (Thorsen et al., 1968)				
Carbon tetrachloride	Water	1.90 to 2.60	1.74	1.99
Ethylene bromide	Water	1.35 to 1.93	1.43	1.64
Methylene bromide	Water	1.34 to 1.71	1.19	1.36
Bromoform	Water	1.31 to 1.71	1.29	1.48
Tetrabromoethane	Water	1.65 to 2.29	2.12	2.44
Ethyl bromide	Water	1.87 to 2.78	1.64	1.88
o-dichlorobenzene	Water	2.3 to 3.51	2.43	2.79
(Krishna et al., 1959)				
n-amyl phthalate	Water	14.0 to 15.3	15.7	18.0
Aniline	Water	7.5	6.44	7.40
m-cresol	Water	6.2 to 7.2	6.72	7.72
Bubbles in liquid (Haberman and Morton, 1954)				
Air	Water	1.24 to 1.90	1.25	1.44
Air	Methanol	0.80 to 1.40	0.88	1.01
Air	Mineral oil	Velocity curve without peak, 4.0 to 6.0 estimated	5.34* N.B.	6.13

$$^* E\ddot{o} = \frac{865(9.81)\,(5.34)^2}{0.0207(10)^6} > 6$$

The lower and upper bounds of D_c are found as follows. At the lower bound $Eö = (\Delta\rho\, g\, D^2/\sigma) \to 0$, Equation (13) becomes

$$D_c = \left[\frac{3\,888\,\sigma}{\rho_c} \left(\frac{\mu_c + \mu_d}{\Delta\rho\, g} \right)^2 \right]^{1/5} = D_{c,\,\mathrm{minimum}} \quad (14)$$

It is evident from Figure 1 that the maximum D_c value that can define a transition point is the value of D at the minimum terminal velocity in regime 2, that is, $U_{2,\,\mathrm{minimum}}$. Differentiation of Equation (11) yields this value; that is

$$D \text{ at } U_{2,\,\mathrm{minimum}} = (6\sigma/\Delta\rho\, g)^{1/2} \quad (15)$$

Therefore, from Equation (15)

$$Eö_{\mathrm{maximum}} = 6 \quad (16)$$

and at the upper bound, from Equations (13) and (16)

$$D_{c,\,\mathrm{maximum}} = 2^{1/5}\, D_{c,\,\mathrm{minimum}} \cong 1.15\, D_{c,\,\mathrm{minimum}} \quad (17)$$

where $D_{c,\,\mathrm{minimum}}$ is given by Equation (14).

Often, minimum and maximum D_c values are close enough to define the transition for practical purposes. Where this is not so, more accurate estimates can be made by substituting D_c values from Equation (14) in $Eö$ in Equation (13) to find D_c.

When $Eö > 6$, transition from regime 1 to regime 2 is not clearly defined.

Experimentally found values of D_c tend to decrease as fluid purity increases (Haberman and Morton, 1954; Thorsen et al., 1968). Most experimental results do not show a sharply defined peak on the U vs. D curve (Figure 1). Therefore, in this work, predicted values of D_c are shown in juxtaposition to the range of D_c values in which $U > 0.9\, U_c$ (Figure 1).

Table 1 shows calculated values of D_c in juxtaposition to reported observed values. The systems shown are within the following ranges of physical properties:

ρ_c : 782 to 998 kg m^{-3}

ρ_d : 1.22 to 2 960 kg m^{-3}

$\Delta\rho$: 17.9 to 1 963 kg m^{-3}

μ_c : $5.2(10^{-4})$ to 0.058 kg m^{-1} s^{-1}

μ_d : $1.7(10^{-5})$ to 0.0185 kg m^{-1} s^{-1}

σ : 0.004134 to 0.0728 kg s^{-2}

If we consider these ranges and that the simplifying assumption that the dissipation rate in regime 1 is entirely due to motion of the interface in viscous fluids and the varying extent of contamination that probably exists in experimental apparatus, the accuracy of predicting D_c is satisfactory.

CONCLUSION

A simplified theoretical approach has been used to provide quantitative prediction of the transition from the viscosity dependent to the viscosity independent regime of free motion of single bubbles and drops. The prediction method invokes all relevant predictable parameters and requires only simple calculations. Accuracy of prediction appears to be satisfactory.

NOTATION

A = area, L^2
C_D = drag coefficient
D = diameter of sphere of same volume as the bubble or drop, L
D_c = D at transition = critical diameter, L
g = gravity acceleration, L t^{-2}
P = power requirement, M L^2 t^{-3}
R = radius of sphere = $D/2$, L
U = terminal velocity, L t^{-1}
U_c = U at transition = U at D_c, L t^{-1}
v_θ = tangential velocity component of potential flow around a sphere, L t^{-1}
V = volume of bubble or drop, L^3

Greek Letters

Δ = difference
θ = angle of latitude, origin at pole
μ = viscosity, M L^{-1} t^{-1}
ρ = density, M L^{-3}
σ = interfacial tension, M t^{-2}
τ = shear stress, M L^{-1} t^{-2}

Subscripts

c = continuous phase μ, ρ
d = dispersed phase μ, ρ
0 = interface

Dimensionless Groups

$Eö$ = $\Delta\rho\, g\, D^2/\sigma$ = Eötvös number
Re = $\rho_c D\, U/\mu_c$ = Reynolds number

LITERATURE CITED

Ackeret, J., "Über exacte Losungen der Stokes-Navier-Gleichungen inkompressibler Flüssigkeiten bei veranderten Grenzbedingungen," *ZAMP*, III, 259 (1952).

Davies, J. T., *Turbulence Phenomena*, Academic Press, New York (1972).

Haas, U., H. Schmidt-Traub, and H. Brauer, "Umströmung kugelförmiger Blasen mit innerer Zirkulation," *Chem. Ing. Tech.*, 44, 18, 1060 (1972).

Haberman, W. L., and R. K. Morton, "An Experimental Study of Bubbles Moving in Liquids," *Proc. Am. Soc. Civil. Engrs.*, **80**, Separ. 387 (1954).

Hu, S., and R. C. Kintner, "The Fall of Single Liquid Drops through Water," *AIChE J.*, **1**, 42 (1955).

Kintner, R. C., "Drop Phenomena Affecting Liquid Extraction," in *Advances in Chemical Engineering*, Vol. 4, p. 52, Academic Press, New York (1963).

Klee, A. J., and R. E. Treybal, "Rate of Rise or Fall of Liquid Drops," *AIChE J.*, **2**, 444 (1956).

Krishna, P. M., D. Venkateswarlu, and G. S. R. Narasimhamurty, "Fall of Liquid Drops in Water," *J. Chem. Eng. Data*, 4, 336 (1959).

Lehrer, I. H., "A Rational Terminal Velocity Equation for Bubbles and Drops at Intermediate and High Reynolds Numbers," *J. Chem. Eng. Japan*, **9**, 237 (1976).

Lehrer, I. H., "Heat and Mass Transfer in Gas-Liquid and Liquid-Liquid Systems, Particle Oscillation Frequencies," *Proc. PACHEC '77*, **I**, 96 (1977).

Lindt, J. T., "Time Dependent Mass Transfer From Single Bubbles," Doctoral thesis, Bronder-Offset N.V., Rotterdam (1971).

Thorsen, G., R. M. Stordalen, and S. G. Terjesen, "On the Terminal Velocity of Circulating and Oscillating Liquid Drops," *Chem. Eng. Sci.*, **23**, 413 (1968).

Tsuge, H., and S. Hibino, "The Onset Conditions of Oscillatory Motion of Single Gas Bubbles Rising in Various Liquids," *J. Chem. Eng. Japan*, **10**, 66 (1977).

Yamaguchi, M., T. Fujimoto, and T. Katayama, "Experimental Studies of Mass Transfer Rate in the Dispersed Phase and Moving Behaviour for Single Oscillating Drops in Liquid-Liquid Systems," *ibid.*, **8**, 361 (1975).

Manuscript received December 4, 1978; revision received February 16, and accepted April 19, 1979.